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List of exchanges with the Publications of the Indian Mathematical Society

India

1. Proc. of the National Institute of Sciences of India.
2. Trans. of the National Institute of Sciences of India.
3. Proc. of the Indian Academy of Sciences.
4. Current Science.
5. Bulletin of the Calcutta Math. Society.

Japan

1. Collected Papers from the Faculty of Science, Osaka Imperial University.
2. Japanese Journal of Mathematics.
3. Journal of the Faculty of Science, Hokkaido Imperial University.
4. Proc. of the Physico Mathematical Society of Japan.
5. Tohoku Mathematical Journal.

England

1. Monthly Notices of Royal Astronomical Society.
2. Proc. of Cambridge Phil. Society.
3. Proc. of the Edinburgh Mathematical Society.
4. Journal of London Mathematical Society.
5. Mathematical Gazette.
6. Math. Notes of the Edinburgh Math. Society.
7. Science Progress.
8. Quarterly Journal of Mathematics.

Sweden

Acta Mathmatica.

Holland

Nieuw Archief Voor Wiskunde.

Belgium

Sphinx.

France

1. Annales de l' Institut Henri Poincare.
2. Annales Scientifiques de l' Ecole Normale Superieure.
3. Bulletin des Sciences Mathematiques.
4. Comptes Rendus de l' Academie des Sciences.

Germany

1. Abhandlungen aus dem math. Seminar Hamburgischen Universität.
2. Deutsche Mathematik.
3. Jahresbericht d. deutschen Mathematiker Vereinigung.
4. Mitteilungen der Math. Gesellschaft, Hamburg.
5. Schriften d. Mathm. Seminars und des Instituts fur Angewandte Mathematik der Univ. Berlin.
6. Gottinger Nachrichten.

U. S. S. R.

1. Bulletin de l' Institut Mathematique et Macanique de l' Universite Kouybycheff de Tomsk.
2. Academie des Sciences de l' RSS d' Ukraine Kiev : Journal de l' Institut Mathematique.
3. Travaux de l' institut math. de Tbilissi, Georgia.
4. Matemakicheskii Sbornik, Moscow.

Rumania

Annales Scientifiques de l' Universite de Jassy.

Poland

Fundamenta Mathematicae.

Czechoslovakia

Publications of the Faculty of Sciences of the Masaryk University, Brno.

16 NOV 1938

THE MATHEMATICS STUDENT

Volume VI] MARCH 1938 [Number 1

THE TENTH CONFERENCE OF THE INDIAN MATHEMATICAL SOCIETY

The Tenth Conference of the Indian Mathematical Society met in Lucknow on the 15th, 16th, and 17th of March 1938, under the auspices of the Lucknow University.

The Address of Welcome to the delegates of the Conference was read by Dr. R. P. Paranjpye, Vice-Chancellor, Lucknow University, on the evening of the 15th March. The Conference was then inaugurated by the Hon'ble Pandit Govind Vallabhai Pant, Premier of the United Provinces. The Secretary of the Society, Dr. Ram Behari, then read the Report of the Society for the years 1936-38. The Inaugural Address was then delivered by Dr. R. Vaidyanathaswami, Reader in Mathematics in the Madras University and Hony. Joint Secretary of the Society.

At 5-30 P.M. Dr. R. P. Paranjpye was at home to the delegates and to the members of the Reception Committee.

At 6-30 P.M. the Hon'ble Justice Sir Shah Sulaiman, Judge of the Federal Court, Delhi, delivered a popular lecture on "The Theory of Gravitation."

On Wednesday the 16th March, the session commenced at 1-30 P.M. and was devoted to the following Symposium Lectures:

Sir Shah Sulaiman: On his own modification of Einstein's Equations.
Dr. A. C. Banerji: Shapes of Nebulae
Dr. V. V. Narlikar: Gravitational Situations
Dr. T. Vijayaraghavan: Tauberian Theorems
Dr. F. W. Levi: Permutations of enumerable aggregates.

At 5-30 P.M. Prof. J. A. Strang, Head of the Mathematics Department, Lucknow University, was "at home" to the delegates and members of the Reception committee.

An hour later Dr. A. C. Banerji, Head of the Mathematics Department, Allahabad University, delivered a popular lecture on "Stars and Galaxies"

On Wednesday the 17th March, a discussion on the teaching of Mathematics in Schools and Colleges commenced at 10-30 A.M.

This was followed by a business meeting of the members of the Indian Mathematical Society to consider suggestions regarding the efficient working of the Society.

The Reading of papers was taken up in the afternoon.

At 5-30 P.M. the Reception Committee was "At Home" to the Délegates.

The popular Lecture for the evening was delivered by Dr. T. Vijayaraghavan.

The conference dissolved on the evening of Thursday 17th March after a hearty vote of thanks to the Chairman and members of the Reception Committee, to the hosts at the several "at homes", to those who had delivered the popular and Symposium lectures and to the volunteers who had helped in the arrangement of, what was acknowledged on all hands, as a very successful session of the Indian Mathematical Conference.

Reception Committee

Chairman

Dr. R. P. Paranjpye, M.A., B.Sc., D.Sc., Vice-Chancellor,
Lucknow University.

Vice-Chairman

Prof. J. A. Strang, M.A., B.Sc., Head of the Mathematics Department
Lucknow University.

Secretaries

Dr. A. N. Singh, M.Sc., D.Sc., Lucknow University.
Prof. N. N. Bose, M.Sc., Lucknow University.

Treasurer

Dr. Lakshmi Narayan, M.A., D.Sc., Lucknow University.

WELCOME ADDRESS

BY

DR. R. P. PARANJPYE, *Vice-Chancellor, Lucknow University.*

GENTLEMEN: I have very great pleasure in welcoming you to Lucknow for the first time for this session of the Indian Mathematical Conference, and I trust that your short stay here will be comfortable and useful. The city of Lucknow is perhaps the most beautiful in the whole country, and although perhaps it cannot boast of an antiquity like Delhi or Benares, it has played a prominent part in the history of India during the last two hundred years. Some of its monuments and buildings will amply repay a visit, even from mathematicians.

2. The University of Lucknow has only a short history and although it cannot claim any very large share in the advance in our science that is going

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on throughout the country, I hope that this visit will encourage us to take our proper share in the development of Mathematics. As in other Universities this subject is taught here up to the M. A. and M. Sc. standards, and we have produced a certain number of students with very good academic record. A few have been engaged on mathematical research and have published a certain number of papers, and I have every hope that the distinction which the University has gained in several other branches of learning will also be soon attained in this branch of Mathematics.

3. Lucknow has been famous in India as the centre of Islamic culture, and although this culture is more in the direction of poetry and letters, still it cannot be forgotten that in ancient days Mathematics formed a prominent part in the history of Hindu and Muslim civilisations. The Arabs, as is well known, formed the connecting link between the Mathematics of the Greeks and the Mathematics of Ancient India, and several Sanskrit works on Mathematics became first known to European scholars through the medium of translations of Arabic renderings of Sanskrit works. It is not, I hope, too much to expect that the present-day culture of Lucknow will be amplified by the addition of mathematical and scientific studies.

4. It is not possible in this welcome address to speak about any abstruse branch of Mathematics, and your subsequent sittings and the lectures arranged will give you ample opportunities of discussing such matters. The Indian Mathematical Society has been doing its best to promote Mathematical studies for over twenty-five years by maintaining a Library and publishing its journals. This work ought, of course, to be continued and extended. But if an *Indian* Mathematical Society is to have any special feature which it is the only competent body to develop, I would respectfully urge that it is in the direction of a detailed study of ancient Indian and Arabic literature in Mathematics. During the last one hundred years many scholars have studied this part of the history of Mathematics, and books on the History of Mathematics like those of CANTOR and BALL devote certain chapters to this subject. But I have always felt that a systematic plan for this purpose is required, and if this Society can formulate that plan and work it strenuously, it will be a great feather in its cap and will, moreover, fill an important *niche* in the history of our science. A member of the Mathematical staff of this University has, in collaboration with another scholar, published the first volume of the History of Indian Mathematics, and I understand that the second volume is soon going to be published. I congratulate the authors on this work, and hope that they will carry it to completion in a short time. I would, however, very much like to see that most of the material for such a history is properly published so as to be available for future workers in the same subject. As is well known, most of the works of the Greek mathematicians are available to the students of the history of Greek mathematics. I should like the same thing in the case of Indian and Arabic mathematics. For this purpose various important works in these languages should be published containing their text, extracts from commentaries on them, their translations, and notes. For bringing out such editions,

the editors should be competent scholars of the languages concerned as well as students of Mathematics. Perhaps, it may not be possible to get many people with this double qualification, and each work might have to be the result of collaboration between two authors, one of whom would be a Sanskrit or Arabic scholar and the other a mathematician. But with the large number of Universities in this country it should not be difficult to get a sufficient number of interested people to undertake this work, which ought to be mainly a labour of love. The editors will have, probably, to do the work without much remuneration. But the expenditure of publishing these editions should be borne by bodies like the Universities, learned Societies, and even perhaps, by Governments in the country. Each volume will probably require not more than three or four thousand rupees, and if two or three volumes are brought out every year, in a few years India will have carried out a work of supreme importance to the History of Mathematics, and have repaid its duty to its ancient *savants*. I do hope that this Society will take the lead in the matter and put itself into touch with other bodies for drawing up a plan for this work. Perhaps, the project I have mentioned may not appear to be exactly within the range of the Indian Mathematical Society, but I have always held the view that the study of the history of a subject ought to be an integral part of the study of that subject itself.

5. I do not propose to delay you any longer. It is a very great pleasure to me to welcome you all here, and in particular to welcome the Hon'ble Pandit Govind Ballabh Pant, the Premier of the United Provinces, who has kindly consented to inaugurate this Conference. Mathematics, perhaps, does not come exactly within the purview of a busy politician and administrator, although even he cannot utterly neglect the subject which has a large bearing on such practical branches of administration as Economics, Statistics, Life Insurance, etc. But no one who has the interest of the people at heart can neglect such an essential avenue of culture as Mathematics. It has always occupied a prominent place in the educational system of every country in the world and will continue to do so in future, in spite of occasional remarks about the alleged waste of time in the study of such a barren subject as Mathematics. If the people of a country are to be properly educated, we claim for Mathematics an essential rôle in that education. I therefore have great pleasure in requesting the Hon'ble the Pandit to inaugurate the Conference.

Message from the Minister of Education, United Provinces

Ministry of Education, United Provinces,
Lucknow.
March 14, 1938.

DEAR DR. SINGH,

I should have very much liked to be present at the forthcoming conference of the Indian Mathematical Society, but I very much regret to say that I have to be out of station during the days the Conference will be in session. I have been a humble student of Mathematics having had the privilege of being a

student of the late Dr. Ganesh Prasad, and try to keep myself in touch, with what success it is not for me to say, with the latest advances in the Science. It is a matter of sincere gratification to me that India's contribution to this branch of Science, a contribution which the world has a right to expect from us, looking to our past, is increasing both in volume and quality as the years pass.

I wish the Conference every success.

Yours sincerely,
SAMPURNANAND.

DR. A. N. SINGH, M. SC., D. SC.,
Secretary, Reception Committee,
Indian Mathematical Society,
Lucknow University.

Secretary's Report for the Period December 1935 to March 1938

BY

DR. RAM BEHARI, M.A., Ph.D. *Hony. Secretary.*

MR. PRESIDENT, LADIES AND GENTLEMEN:

1. On behalf of the Managing Committee of the Indian Mathematical Society, I have the proud privilege to extend to you all, a hearty welcome to this Conference. We are profoundly grateful to the members of the Society and to other distinguished mathematicians, who have come from all parts of the country to attend this Tenth Conference of our Society, which is intended to promote the advancement of Mathematical study and research. We are equally grateful to the many citizens of this historic city of Lucknow, who have responded to our invitation by gracing this occasion with their presence.

We are specially grateful to you, Sir, for the trouble you have taken, inspite of your multifarious duties to come to inaugurate our deliberations.

Lastly I take this opportunity of giving expression to the grateful thanks of the Society to the various Local Governments, Native States, Universities and Institutions for the facilities they have granted to their officers to enable them to attend this Conference.

2. First of all it is my painful duty to record the heavy loss which the Society has sustained in the deaths of:

(i) V. Ramaswami Aiyar, M.A., who founded the Society in 1907 and rendered valuable services in various capacities; who was a great source of inspiration to our younger members, and whose love and devotion to the subject is embodied in the large number of interesting papers, notes and questions, that have appeared in our Journals. Those who attended the last Conference held at Delhi must be remembering him well.

(ii) Dewan Bahadur R. Ramachandra Rao, B.A., an ex-president of the Society for two years (1915-1917), who was instrumental in discovering and helping the late Srinivasa Ramanujan, F.R.S.

(iii) Rao Bahadur S. Narayana Iyer, M.A., who was an enthusiastic member of the Society from its very inception, Assistant Secretary (1907-1910); Honorary Treasurer (1914-1928); It was he who helped in securing Ramanujan a post in the Port Trust Office which gave him facilities to continue his research.

(iv) Prof. V. B. Naik, M.A. of Fergusson College, Poona, Honorary Assistant Librarian (1907-1922); Honorary Librarian (1922-1937), through whose selfless work and paternal care the Library of the Society grew considerably.

(v) Prof. S. Mukhopadhaya, M.A., Ph.D. of Calcutta University, who took an active interest in the affairs of the Society, and was a member of the Managing Committee at the time of his death.

3. Turning to routine matters, during the time that has elapsed since we last met at Delhi in December 1935, about 30 new members have been enlisted, including one Life Member. Prof. G. N. Watson, Prof. E. H. Neville and Dr. Vaidyanathaswamy have during this period kindly accepted our invitation to be enrolled as Honorary Members of the Society.

4. The Society continues to publish two journals, each of which appears four times a year. The first, which retains the appellation, the 'Journal of the Indian Mathematical Society,' publishes only original papers and caters for the needs of workers in the field of advanced Mathematics. It is under the fostering care of Dr. Vaidyanathaswami of the Madras University. The other one under the title 'The Mathematics Student' is edited by Prof. A. Narasinga Rao of the Annamalai University and meets the requirements of Students and Teachers of Mathematics in Colleges, and of young research workers.

5. The Library of the Society is flourishing under the care of Prof. R. P. Shintre and Prof. D. D. Kosmbi, both of the Fergusson College, Poona. A most welcome feature is the increase in the number of journals received in exchange. In 1933, their number was only 29, while now it has gone up to 62. This change has been brought about by the efforts for our Librarians and the co-operation of various mathematicians abroad. In particular, we acknowledge our gratitude to Prof. Tullio Levi-Civita of the University of Rome, who gave a great deal of his precious time and used all his influence in having the *Rendiconti* of the Lincei Academy exchanged with our publications.

The number of journals subscribed by the Library has also increased appreciably. Some important publications like the *Mathematische Zeitschrift*, *Zentralblatt für Mathematik*, *Composito Mathematica*, have been recently added to the list.

Due to the increase in the number of journals, the space which the Library Hall of the Fergusson College could afford, proved insufficient for the needs of the Library. The books were, therefore, shifted to a room on the ground floor

of the same building, open steel shelves extending up to the ceiling were specially constructed, and space has been provided not only for the journals in stock, but also for those likely to come in for several years more.

6. Regarding the circulation of journals, I take this opportunity of soliciting the co-operation of the members who wish to take advantage of the circulation privilege in making this part of the work of the Library free from delays, mistakes and losses.

7. With regard to the financial position of the Society, every year several members are taking advantage of the reduced rate of life composition to become life members, with the result that the annual income from ordinary membership fees is gradually going down. The Managing Committee hopes that with the willing co-operation of all the members, the deficit will be made up by enlisting new members, by persuading members in arrears to pay up promptly, and by enlisting more institutional subscribers for the journals. So far the Government of India have not seen their way to grant us the financial assistance sought from them. The Committee hopes that in view of the Society's pressing needs for the housing and funds for the library, the Government of India and the Universities will come to our aid by making suitable annual grants.

8. This Conference should in the usual course have been held during the last Christmas, which perhaps, would have suited our members, especially those from the South better, but in view of the Session of the Indian Science Congress early in January, it was not considered suitable by our hosts. It will serve to show the interest evinced by our Members in these periodical Conferences if I mention that the number of papers has grown steadily from 13 read in the first Conference held at Madras, to 40 in the last Conference held at Delhi. This time, the number of papers contributed is even more. We have also been able to arrange Symposium lectures on different research topics.

9. In conclusion, the Committee, once again, takes this opportunity to express its feelings of thankfulness for the trouble taken by you, Sir, to come here in the midst of your onerous duties, to initiate the delibrations of this Conference. The Committee also has great pleasure in offering its thanks to all those who have been ungrudgingly contributing their mite for the welfare of our Society, during these 31 years of its existence, and especially to those who have undertaken long and tedious journeys to attend this Conference and to contribute towards its success.

The Managing Committee also thanks the various Indian Universities, that have rendered financial help in the past by giving annual grants-in-aid.

My last but not the least duty is to express, on behalf of the Managing Committee, our grateful thanks to the Vice-Chancellor of the University of Lucknow, for giving us this opportunity of assembling here under the auspices of the University by his kind invitation, and to the Local Reception Committee for their hospitality and excellent arrangements for the guests.

Our special thanks are due to Dr. A. N. Singh and his band of volunteers who have worked hard to make this Conference a success.

STARS AND GALAXIES

BY

PROF. A. C. BANERJI, D.SC., *Allahabad.*

(*Popular lecture delivered at the Mathematical Conference at Lucknow on 16-3-38*)

Gentlemen, You have seen stars and nebulae or at least you are aquainted with them. You may also have observed the Milky-way on a dark clear night. It appears as a faintly luminous band forming an arch which stretches across the sky from end to end. If you see it through a good telescope you will perhaps think that it is composed of "countless multitudes" of stars. But the number is not really uncountable. The total number is of the order of two hundred thousand millions *i, e.*, 2×10^{11}. Each star is an immense fiery globe similar to our Sun. The number of human beings on our earth is about two thousand millions. So the number of starry inhabitants in the Milky way is about two hundred times the human population on our planet. All these stars form a system or a galaxy whose size is enormous but not unlimited. As we are situated within the system, we cannot get a good view of its form. An inhabitant of Andromeda Nebula, or more generally a person situated outside the Milky-way will see it as a little island of matter in an ocean of empty space, just as we find the Andromeda nebula to be a tiny island in the ocean of emptiness. Or we may put it in a slightly different way and say that both the Milky way and the Andromeda nebulae are oases in the desert of emptiness. Although they are mere oases in empty space, their sizes are enormous compared to us. To measure their dimensions it is most inconvenient to employ our ordinary units of distances, and we have to use instead our astronomical units. The usual units that are used are one light year and one parsec. One light year is the distance covered in one year by light travelling with the speed of 186,000 miles per sec. One light year is roughly equal to 6 billion miles (6,000,000,000,000 miles). One parsec is the distance of a heavenly body whose parallax is one second. *i. e.*, it is about 20 billion miles (20,000,000,000,000 miles).

According to most recent estimate by Hubble and Trumpler, the diameter of the Milky way is about 30,000 parsecs. Lindblad's theoretical treatment gives 26,000 parsecs as the diameter. Our galactic system or the Milky way is a greatly flattened disc with the thickness at the centre of about 6000 parsecs, Our sun is at a distance of 10,000 parsecs from the centre of mass of the galactic system. This centre is situated in the Constellation of Ophiuchus. By studying the motions of stars in our galaxy, it has been found that our galactic system is a dynamical unit revolving in its own plane round the centre like a huge Catherine wheel. The period of rotation is about 220,000,000 years, and the tangential velocity at the

sun is about 200 miles per sec. (275 Km/Sec.). The total mass of the galaxy is about 16·5 × 10^{10} times the mass of the sun. The largest known external system (external to our galaxy) is the Andromeda nebula. According to most recent estimate it has a diameter of about 20,000 parsecs. Shapley thinks that it is also a greatly flattened stratum of about 1000 parsecs in thickness except near the galactic centre where it is greatly enlarged. Its period of rotation is 14,000,000 years. Its mass is about 3·5 × 10^{9} times the mass of the sun. Hubble has shown the practical identity of the constituent members of the Andromeda nebula and our galaxy. Each contains stars and star clouds, bright and dark nebulae, diffuse nebulae, globular clusters, cepheid variables novae, giant and super-giant stars. The similarity of structure of the two systems is strongly indicated by their greatly flattened forms. Both are in rapid rotation in their planes and this rotation is perhaps the cause of their flattening The Andromeda nebula has the form of a spiral and it is not unreasonable to suppose that our galaxy is a great·spiral nebula, perhaps somewhat larger than any of the extra galactic system, but not of a different order of size. The Andromeda nebula and the Milky way are super galaxies, and several super galaxies form together a meta galaxy. If we take the analogy of terrestrial geography then a *meta galaxy* may be compared to a *continent* and a *super galaxy* to a *country* within a continent. "Know thy country well" is the slogan at present and we shall therefore study more in detail the constituents of our galactic system *i. e.*, the Milky way.

I already mentioned to you that the Milky way contains "multitudes of stars". These stars individually show a very wide diversity. Some radiate more than 10,000 times as much heat and light as our sun while some others give out less than 1/100th times as much as our sun. Some show a surface temperature as 30,000° C. while others have got surface temperature as low as 3,000° C. These stars have widely varying degrees of apparent brightness due (i) partly to their distances and (ii) partly to their actual differences in intrinsic brightness.

Some stars are believed to be pulsating. They undergo inflation and deflation within a period of a few days or a few weeks. They give out widely varying quantities of heat and light as they swell up and collapse. A great many stars, about ⅓ of the whole number, travel in pairs and they are called double or binary stars, but the majority of them to quote the language of Eddington, are "bachelors" like our sun. Some stars are very dense and compact, while others are extremely tenuous. The stars also vary enormously in size.

But in spite of their diversity in many respects there is some sort of uniformity amongst the stars so far as their masses are concerned. But for a few exceptional ones the masses of stars vary within a range from 1/5th to 5 times the sun's mass. The general run of masses is even within a narrower range. Adult persons have widely different features, but their masses will generally vary within the narrow range of 1 md. and 2½ mds.

A giant star is big not on account of its mass but because it is swollen up like a balloon. Some of these stars are as rarefied as gas. A dwarf star is

small because it is highly compressed. Our sun is a typical middle class star. The mass of our sun is $1{\cdot}99 \times 10^{33}$ gm., its radius is $6{\cdot}9 \times 10^{10}$ cm. and its mean density is 1·4 times that of water. The surface temperature of the sun is about 5000° C. whereas the temperature at the centre may be as high as 14,000,000° C.

Capella which is the brightest star in the constellation Auriga is a binary star. Its bright component has a mass $8{\cdot}3 \times 10^{33}$ gm. or $4{\cdot}18 \times M$ where M is the mass of the sun. Its radius is $9{\cdot}55 \times 10^{11}$ cm. or $13{\cdot}74 \times R$ where R is the radius of the sun. Its mean density is ·00227 gm. per cu. cm. So Capella has an average density about the same as the air in our atmosphere. Its surface temperature is about 5200° C. while the temperature at its centre is estimated to be over 7 million degrees (7,000,000° C.).

Betelguese is a real giant star in the constellation of Orion. To get an idea of its size we have to imagine our sun to swell up and extend up to the orbit of Mars, swallowing up Mercury, Venus, and the Earth. Its radius is $2{\cdot}5 \times 10^{13}$ cms. or $360 \times R$. The mean density is ·000002 gm. per cu. cm. Therefore the matter in Betelguese is extremely tenuous. It is as rarefied as the gas in an exhausted vessel for instance--a vacuum tube. For terrestrial conditions, Betelguese may be regarded as "a rather good vacuum", but when we consider the extreme emptiness of space Betelguese can no longer be regarded as a vacuum, it then becomes an aggregate of matter—a star. Recently it has been possible to produce a more highly exhausted vacuum than Betelguese in our laboratories. If a star of the size of Betelguese has as high a density as the sun, then the force of gravity will be so great that light would be unable to escape from it and rays would fall back to the star like a stone to the earth.

In gaseous nebulae we observe matter which is a million times more rarefied than the most highly exhausted vacuum obtainable on earth. In our laboratories we can hardly produce temperatures more than 5,000° C., but in stars we some time come across temperatures as high as 50,000,000° C. Eddington has very aptly stated that in the laboratories of the heavens we come across much hotter furnaces and much more rarefied "vacuum"; and a vaster scale of experiments is in progress there.

White dwarf Stars

Eddington has compared an astronomer to an electric engineer. The electric engineer will tell you that to produce a certain amount of illumination, you must have a dynamo of a specified size. Similarly it is possible for an astronomer to calculate the mass of a star which gives out a certain amount of light and heat. Eddington thus found the "mass-luminosity law" of a star *i.e.*, the intrinsic brightness of a star depends mainly on its mass and to a small extent upon its radius. This formula was not intended to apply to all stars but only to diffuse stars which behave like perfect gases. The observed masses and luminosities of diffuse stars agreed with the formula all right, but the trouble arose when dense stars for which the formula was not

intended, agreed with the formula equally well. This surprising result needed explanation. It meant that although the density was as great as water or iron the stellar material was behaving like a gas and was compressible like an ordinary gas. We have to look into this point more carefully. Let us see why we can compress air and not water to any appreciable extent. It is because that the air molecules are far apart with plenty of empty space between them. When air is compressed the molecules are packed a bit closer, then the amount of vacant space between them is reduced. In liquids and solids the molecules are practically in contact and the limit of compression has already been reached so that the particles cannot be packed closer together any further; and the densities of these liquids and solid substances can no longer be increased appreciably. So under terrestrial conditions the limit of compression corresponds approximately to the density of the liquid or the solid state.

But stellar conditions where temperatures of millions of degrees are reached, are very different from conditions prevailing on our earth. At such high temperatures atoms are thoroughly smashed up or ionised. Under ordinary terrestrial conditions an atom has a heavy central nucleus surrounded by widely extended but not very substantial structure of revolving electrons. At the high temperature prevailing in the stars this structure of electrons is broken up and they are scattered loose. Vast extent of empty space is now available; the atoms do not jam, and the limit of compression is not reached until a density very much greater than that of water or indeed of any material found on this earth, is attained. Stellar matter is far from the limit of compression which has been put enormously higher due to the smashing up of the outer parts of the atom. So this matter behaves like a perfect gas.

The companion of Sirius, the brightest star visible in the sky, has a density 50,000 times that of water—the matter in this star is 2000 times as dense as platinum. If a match box is filled with this matter, it would weigh a ton. The star O_2 Eridani B has a density about 98,000 times that of water.

Planetary Systems

At one time it was thought that all stars have attendant planets. Now the opposite view is taken and it is supposed that planetary system is a rare development and a "freak-formation." The formation of planets of our sun is a rare accident. It is surmised that some two thousand million years ago our sun nearly collided with another star. There was no actual smash. The second star has passed by and is far away now. A great tidal wave was created in our sun at the time of the closest approach, and jets of matter were raised and thrown out, which have condensed into planets now. There is immense room in inter-stellar space, and Eddington has rightly pointed out that notwithstanding the reckless speed of stars, the celestial traffic conditions are extremely safe, and collisions are very rare occurrences. It has been calculated that with the present distribution of stars in the neighbourhood of the sun, a given star is likely to meet with actual collision once in 6×10^{17} years.

If we allot an average age of 5×10^{12} years to the stars, we find that only about one star in 100,000 is at present surrounded by planets.

Binary Stars

The two constituents of some of the binary stars are so closed together that they cannot be differentiated by direct visual observation. Only their spectra reveal that they are binary stars. They are called *spectroscopic binaries*. They are susceptible to direct visual observation. It is surmised that spectroscopic binaries are formed out of rotating masses of nearly uniform density.

The parent rotating mass which had originally a spherical shape assumes first the spheroidal and then the pear-shaped form through shrinkage. The pear-shaped form is dynamically unstable and ultimately fission occurs, and two detached masses are formed, which make a binary system.

On the other hand the visual binaries are formed out of independent condensations in the parent nebulae.

Cepheid Variables

Cepheid Variables are perhaps pulsating stars. They are very useful in measuring the distance of remote stars and nebulae. Parallactic method of measuring distances fails when a heavenly body is at a distance greater than 100 parsecs. Brightness or luminosity of a Cepheid variable undergoes continuous change periodically, and the period varies from a few hours to a few weeks. Cepheids which have the same period, have the same intrinsic luminosity, radius and spectral type. The observed relation between period and luminosity is governed by *period luminosity law*. The intrinsic luminosity of the Cepheids varies more or less as the period. So Cepheid Variable can be made to act as "Standard candles." A Cepheid having a period of 40 hours has an intrinsic luminosity 250 times that of the sun and a Cepheid having a period of 10 days has got a luminosity 1600 times that of the sun. When the absolute and apparent magnitudes of a Cepheid are known a simple application of the inverse square law will give its distance. If A and B have the same intrinsic brightness, then if A looks four times as bright as B, B must be at a distance twice that of A.

Luckily for us, most of the star clusters and nebulae abound in Cepheid Variables, and so their approximate distances are easily found.

Planetary and diffuse nebulae

Planetary nebulae have nothing of a planetary nature in them, but they appear to have finite discs when seen through a telescope. These nebulae contain a great number of stars. They are very much rarefied. Eddington has calculated that the whole amount of matter contained in a volume of the size of the earth would not weigh more than 20 tons.

Diffuse nebulae are irregular in shape. Variation of density, opacity, and luminosity is responsible for all sorts of fantastic shapes. In our galactic system we come across both these types of nebulae. Some of them have a diameter of 100 light years or more. If our galactic system is compared to a *country* on the same analogy these nebulae can be compared to *provinces* on our terrestrial map.

There are also dark nebulae which do not shine and which obscure the stars which lie behind them. The Pleiades and the Hyades are galactic clusters which are much smaller in size. The length of the Pleiades is about 10 light years from end to end. These galactic clusters may be compared to *districts* and *divisions*. On the same scale the sun with its planets may be compared to a *city* with *its suburbs*.

There is another kind of cluster called the globular clusters which are much richer in stars than galactic clusters.

Magellanic clouds and extra-galactic nebulae

Just outside the boundary of our galaxy there are two remarkable groups of stars which were first observed by the famous Spanish Explorer Ferdinand Magellan near the South celestial pole when circumnavigating the world. Their distances are about 85,000 and 95,000 light years from us. In the depths of space there are many extra-galactic systems or nebulae, and out of these a large number have spiral forms while others have got elliptical shapes. The great nebula Andromeda is about 800,000 light years from us. Many of these extra-galactic nebulae have got enormous size. If the size of some of these nebulae be reduced to that of Asia, then the Earth would become a microscopic body hardly visible even under a most powerful microscope. Many of the spiral nebulae have spiral forms.

Extra-galactic nebulae abound in Cepheid Variables and their distances can therefore be easily measured. We notice one remarkable thing about these extra-galactic nebulae. Except for three or four of the nearest of them their spectra show a shift of spectral lines towards the red. It is moreover found that on the average the amount of the shift is directly proportional to the distance of the nebulae. Perhaps the most probable explanation is to be given by Doppler's Principle. In accordance with this principle we find that these galaxies are running away from us and the further they are away, the faster they go. The speed is found to be proportional to the distance, so that a galaxy 10 million light years away recedes at the rate of 1000 miles per second, a galaxy over 50 million light years away recedes at the rate of 5000 miles per second. The fastest yet discovered recedes at the rate of 30,000 miles per second. Its distance is 300 million light years away from us. In miles, 300 million light years = 1800,000,000,000,000,000,000 miles.

It may be mentioned here that in the case of a nebula we cannot get a good clear spectrum like that of a star. This is due to the fact that a nebula

consists of multitudes of stars with all sorts of different spectra which are superposed. We therefore get a sort of a blur in which two main features common to all the stars are shown prominently. These are H and K lines of Calcium, and the shifts of these lines are measured, from which we can deduce the velocity of recession of the nebulae.

The supporters of the theory of Relativity have tried to explain the phenomenon of recession of galaxies by suggesting that it is due to general scattering caused by *cosmical repulsion*. In all small scale systems gravitation gets the upper hand whereas in a large enough scale cosmical repulsion preponderates. In accordance with this theory the nebulae will recede to double their present distance in 1300 million years.

There is one point which I would like to mention to you. We can only know ancient, indeed, very ancient history of the nebulae. A nebula whose distance is observed to be 10 million light years from us was at that distance 10 million years ago. At the present moment it is somewhat further away i.e., about 10,050,000 light years away from us, and is receding from us at the rate of 1050 miles per sec. Similarly a nebula whose distance is observed to be 300 million light years from us was at that distance 300 million years ago. At the present moment it is about 350 million light years from us and is receding now at the rate of 35,000 miles per second.

I am indebted to Eddington, Jeans, Shapley, Hubble, Lindblad and others for borrowing data from their papers in preparing this lecture.

ABSTRACTS OF PAPERS

F. W. LEVI Hardinge, Professor of Mathematics,
Calcutta University

Permutations of enumerate sets

P. SAMUEL LALL, Punjab University

Dedekind's Sections of real numbers and Mahajani's variation of it.

S. M. SHAH, Muslim University, Aligarh

An inequality for an arithmetical function.

S. S. PILLAI, Annamalai University

'*On Waring's Problem*' $g(6) = 73$.

As usual, let $g(n)$ denote the least value of s such that every number is the sum of s n^{th} powers. Then for $n \leqslant 100$, we do not know the exact value of $g(n)$ only when $n = 4$, 5 and 6. In this paper I show that

$$g(6) = 73$$

The proof rests on certain results quoted by L. E. Dickson from a table prepared by R. Shook. While no table is required to determine $g(n)$ when $n \geqslant 7$, we want Shook's extensive table to determine $g(6)$. Though a proof which does not depend upon an extensive table is highly desirable, the author wonders whether it will be possible unless one finds out a method which is entirely different from all existing ones.

K. SAMBASIVA RAO, M.A. (Hons.), Andhra University.

On a particular representation of integers as sums of kth powers.

We represent an integer x in the form $x = x_1^k + x_2^k + \cdots x_s^k$ where $x_1\, x_2 \cdots x_s$ are integers given by

$$(x_1+1)^k > x \geqslant x_1^k\,;\ (x_2+1)^k > x - x_1^k \geqslant x_2^k \text{ etc.}$$

If the number of kth powers required for representing x is $S_k(x)$ it is shown that

(i) for all $x > x_0$, $S_k(x) < c_1 \log\log x$;

(ii) there exist infinitely many x for which $S_k(x) > c_2 \log\log x$.

HANSRAJ GUPTA, Government College, Hoshiarpur.

Congruence Properties of Self-Contained Balanced Sets.

A set B_k of positive integers $b_1, b_2, b_3, \ldots, b_k$ is called a balanced m-set, when the sum of the b's equidistant from the beginning and the end is m. It is said to be self-contained modulo m, when numbers ξ exist such that the members of set ξB_k, are congruent modulo m with the members of set B_k, each to each though not necessarily in the same order.

Denoting by $G_r(B_k)$ the sum of the products taken r at a time of the members of set B_k, and by $S_r(B_k)$ the sum of the rth powers of b's, the following results are proved :—

$$2\,(m,\ M_{2j})\ G_{2j+1}\,(B_k) \equiv 0 \pmod{m^2},$$
$$\text{and } 2\,(m,\ M_{2j})\ S_{2j+1}\,(B_k) \equiv 0 \pmod{m^2}.$$

where M_{2j} is a certain product quite independent of m.

HANSRAJ GUPTA, Hoshiarpur

A Generalisation of Leudesdorf's Theorem.

Denoting by G_r the sum of the products taken r at a time of numbers less than and prime to m, the following generalisation of Leudesdorf's Theorem is proved :

$$2\,(m,\ 4N_{2j})\ G_{2j+1} \equiv 0 \pmod{m^2}$$

where N_{2j} is the product of all odd primes of the form $(2j/d+1)$.

HANSRAJ GUPTA, Hoshiarpur

Waring's Problem for powers of Primes.

In this paper, decompositions into 12th powers of primes are considered. The weight of any integer $\leqslant 11^{12}$ can be found with the help of tables given. Finally it is shown that the weight of no number between 7^{12} and 10^{48} exceeds 1670.

D. R. KAPREKAR, Devlali

Demlo Numbers and their applications to recurring decimals.

A Demlo number is an integer whose digits may be divided into three groups of which the middle group consists of a repeated digit and the sum of the number given by the two extreme groups is also a number with the same repeated digit. Thus 12488764 is a demlo number since $124 + 764 = 888$. Methods of deriving such numbers, their properties and relation to recurring decimals are discussed. A special class of such numbers are those in which the digits go on increasing and then decrease e.g. 123454321. They have some striking properties ; thus

$12345678987654321 \times \mathbf{243} = \mathbf{2999999994}000000003$.

A. A. KRISHNASWAMI AYYANGAR, M.A., L.T.
Maharaja's College, Mysore.

A New Study of the half-regular continued fraction.

In this paper it is proposed to consider the structure and properties of the several types of half-regular continued fractions through a study of the transformations which convert a simple continued fraction into a half-regular one. This method enables us to give a synoptic survey of known results and many new ones concerning the half-regular continued fraction. The discussion runs into *five* sections:

(1) Sets of Transformations. (2) Properties of Residues.
(3) Varieties of Types. (4) Raschesten Kettenbruchentwickelung.
(5) Recurring continued fractions,

the main aim being to bring together as many results as possible under one unifying principle.

T. VENKATARAYUDU, M.A., Andhra University, Waltair.

The Algebra of the eth power residues.

In the present paper the algebra of the Gaussian periods is shown to be identical with the algebra of the eth power residues mod. p. The latter is easily seen to be a particular case of the algebra of the eth power residues connected with a cyclic group of finite order. Further an example is given to show that there is no corresponding algebra of the eth power residues connected with any finite Abelian group.

S. PANKAJAM, University of Madras.

On finite Boolean Algebra

This paper is concerned with a proof of the known theorem that any *finite Boolean Algebra can be regarded as an algebra of all sub-classes of a finite class.* The proof, though not new in principle, has, I believe, some novel features. Further, I also discuss the representation of a finite Boolean Algebra as a Vector Algebra, (mod. 2).

C. N. SRINIVASIENGAR

Non-differentiable functions

M. N. NARASIMHA IYENGAR, Mysore

A General form of the Taylor-Remainder:

It is proposed to show that by utilising Cauchy's form of the mean value theorem it is possible to obtain a general form of the

Remainder in Taylor's theorem, which includes as particular cases all the forms of the Remainder known so far, including the Integral forms. A short note on the behaviour of 'θ' is also included.

M. N. NARASIMHA IYENGAR, Mysore

'*Some Inequalities connected with the Error-Function Integral and the Gamma Integral.*'

The approximate position of the Centre of Gravity of the area is used to derive these inequalities. One of them is

$\frac{4}{a}\,\frac{1-e^{-a^2/2}}{1+e^{-a^2/2}} > \int_0^a e^{-x^2/2}dx > \frac{2}{a}\,(1-e^{-a^2/2})$. This inequality can be made stronger: As it is, the portion $\int_0^a e^{-x^2/2}\,dx > \frac{2}{a}(1-e^{-a^2/2})$ seems to be better than $\int_0^a e^{-x^2/2}dx > \sqrt{\pi/2}(1-e^{-x^2})\;(1 > x > 0)$ in the interval $(0, \cdot 95)$.

M. N. NARASIMHA IYENGAR, Mysore

'*Some simple properties of the Logarithmic curve and their applications.*'

The locus of the mid-points of a system of parallel chords is obtained and it is shown how all the variables that occur in a paper by Prof. G. N. Watson on 'the approximations connected with e^x' (published in the Proc. L. M. S.) can be interpreted geometrically.

M. N. NARASIMHA IYENGAR, Mysore

'*A theorem stated by Ramanujan*'. (Approximations connected with e^x).

The following theorem was stated by Ramanujan:

$1+\frac{x}{1\,!}+\cdot+\frac{x^x}{x\,!}y=\frac{e^x}{2}$, where $y=\frac{1}{2}$ when $x=0$ and $y\rightarrow 1/3$ when $x\rightarrow\infty$ and he remarked that it is difficult to prove that y lies between $\frac{1}{2}$ and $\frac{1}{3}$. Professors Watson and Szego have published proofs in *Proc. L. M. S.* and *Jour. L. M. S.* respectively. In this paper an attempt is made to give an elementary proof of the same result.

V. GANAPATHY IYER, Madras

"*On the average radial increase of integral functions of order one and finite type.*"

Let $z=re^{i\theta}$. Let $f(z)$ be an integral function and $M(r, f)=\max\limits_{|z|\leqslant r}|f(z)|$. We denote by $n(r)$ the number of zeros of $f(z)$ in $|z|\leqslant r$

and by $n(r, \alpha, \beta)$ those in the region $\alpha \leqslant \theta \leqslant \beta$, $|z| \leqslant r$. The function $f(z)$ is said to be of order one and finite type when $\overline{\lim\limits_{r \to \infty}} \dfrac{\log M(r, f)}{r} < \infty$. In this paper I prove the following result:

THEOREM. Let $f(z)$ be of order one and finite type and let

$$\int_{1}^{\infty} \{\log^{+}|f(x)| + \log^{+}|f(-x)|\} \frac{dx}{x^2}$$

be convergent, where $\log^{+}|a| = \max(0, \log|a|)$. Then the following relations are valid:

(i) $\lim\limits_{r \to \infty} \dfrac{n(r, \delta, \pi - \delta)}{r} = \lim\limits_{r \to \infty} \dfrac{n(r, -\pi + \delta, -\delta)}{r} = 0$ for $0 < \delta < \pi$;

(ii) there exists a constant A such that $\lim\limits_{r \to \infty} \dfrac{n(r)}{r} = A$;

(iii) $\lim\limits_{r \to \infty} \dfrac{n(r, -\delta, \delta)}{r} = \lim\limits_{r \to \infty} \dfrac{n(r, \pi - \delta, \pi + \delta)}{r} = \frac{1}{2} A$ for $0 < \delta < \pi$;

(iv) $$\lim_{R \to \infty} \frac{1}{\log R} \int_{1}^{R} \left\{ \log|f(t e^{i\alpha})| + \log|f(t e^{i(\alpha - \pi)})| \right\} \frac{dt}{t^2} = \pi A |\sin\alpha| \text{ for } 0 \leqslant \alpha \leqslant 2\pi.$$

Results (i), (ii) and (iii) have been proved by Levinson* and M. L. Cartwright.† The relation (ii) has also been proved by Paley and Wiener.‡ The result (iv) is new.

R. S. VARMA, Cawnpore

Some Infinite Integrals involving Weber's parabolic cylinder functions.

The object of this paper is to evaluate some infinite integrals involving Weber's parabolic cylinder functions. Two functions of the parabolic cylinder are also obtained which are J_n—transforms of each other. Finally some integral expressions for the Humbert function $J_{m, n}(x)$ are investigated.

R. S. VARMA, Cawnpore

On Laguerre Polynomials which are self-reciprocal in the Hankel transform

In this note some Laguerre polynomials which are self-reciprocal in the Hankel transform have been investigated.

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* *Proc. Cambridge Phil. Soc.*, 31 (1935), 335-46.

† Ibid., 347-50 and the *Proc. Lond. Math. Soc.*, (2), 38 (1935), 179.

‡ "Fourier Transforms in the Complex Domain", *Amer. Math. Soc. Colla. Pub*, 19 (1934), 69.

B. RAMAMURTI, Annamalai University

A special web of binary sextics

The object of this paper is to study the special web of binary sextics

$$A\,\beta_x^2\gamma_x^2\delta_x^2 + B\,\alpha_x^2\gamma_x^2\delta_x^2 + C\,\alpha_x^2\beta_x^2\delta_x^2 + D\,\alpha_x^2\beta_x^2\gamma_x^2$$

where $\alpha_x^2\ \beta_x^2\ \gamma_x^2\ \delta_x^2$ are general quadratics, by means of a representation in [3] with reference to a twisted cubic. Surprisingly enough it leads to the net of quadrics associated with a pair of Mobiüs tetrads, which has been studied in detail recently by W. L. Edge (*Proc. London Math. Soc.*, 1936). Some important properties of such a net obtained by him, and some others are deduced in this paper by binary methods.

A. NARASINGA RAO, Annamalai University

Oriented Turbine Geometry

The oriented turbine is a generalisation of the concept of the oriented sphere and is a configuration of oriented surface elements obtained as follows :—Let the surface elements of an oriented sphere (inner sphere or hub of the turbine) in n-dimensions glide along the tangent prime in all directions through a constant distance t from the point of contact. The points of the resulting configuration of oriented surface elements lie on a sphere (the outer sphere or rim of the turbine) and make with its surface elements a constant angle α. When the rim and the hub of a turbine coincide we have an oriented sphere which is thus a special kind of turbine. It is shown how the Lie Group in Hohere-Kugelgeometrie may be considered as a Geometry of Turbines under a contact transformation group, and how not only the Geometries of Mobius and of Laguerre but a whole pencil of sub-geometries of Lie which belong to the Mobius-Laguerre pencil may be imbedded in Turbine geometry.

A. NARASINGA RAO AND K. RANGASWAMI,
Annamalai University

On the Pedal and Contact Circles of a Triangle

A pair of isogonal conjugates P, P′ with respect to a triangle define two circles—their "pedal" and "contact" circles. The former passes through the projections of these points on the sides, while the latter is the circle through the points of contact of the two in-conics with centres at P and P′. In this paper these circles are studied through the usual representation of the circles in a plane by the points in a three-space.

It is known that the totality of pedal circles and also that of contact circles correspond to points on quadrinodal cubic surfaces, the in- and ex-circles of the triangle corresponding to the nodes. It is shown that the collineation which carries the contact circle of a pair isogonal conjugates into the pedal circle of the same pair is the one which has the in- and ex-circles of the triangle for fixed elements and carries all straight lines in the plane into circles orthogonal to the polar circle of the triangle.

A. NARASINGA RAO AND B. R. VENKATARAMAN,
Annamalai University

The infinitesimal flow in a projective space

When an infinitesimal projective transformation is conceived as a flow of points on the plane supposed endowed with a Euclidean metric, it is found that the divergence and curl of the flow vector at a point P are proportional to its distances from two fixed lines at right angles passing respectively through the centroid and ortho-centre of the invariant triangle. Taking the two lines as the x and y axes, it is found that the velocity field may be resolved into (i) a constant translatory motion, (ii) that due to a homogeneous strain of zero divergence and (iii) a radial outflow along OP with a velocity ry where O is the origin and $OP = r$.

The projective flow in Euclidean n-space shows the same features, the Stokes tensor here taking the place of the curl of the vector field.

C. N. SRINIVASIENGAR

On the Quartic Developable

Deals with a family of Cayley cubic Scrolls associated with a quartic developable, all of whose generators belong to the same linear congruence.

The first polar surface of the developable with respect to any point on a generator is a cubic of the type (B_5, C_2), whose asymptotic curves are determined.

RAM BEHARI, University of Delhi

Some properties of the line of striction of a ruled surface

The object of this paper is to obtain both geometrical and analytical proofs of some properties of the line of striction of a ruled surface, and to give an easy method for constructing a ruled surface whose line of striction is any given curve.

RAM BEHARI, University of Delhi

On the generators of a ruled surface

Mainardi-Codazzi relations are generally employed by writers to obtain some properties of the generators of a ruled surface. In this paper I have obtained an easy method for proving some properties without any reference to Maindardi-Codazzi relations.

D. D. KOSAMBI, Fergusson College, Poona

Differential invariants connected with systems of partial Differential Equations: By using the methods followed in my work on ordinary differential equations, it is possible to obtain the differential invariants of the system: $\frac{\partial^q x^i}{\partial u^{\alpha_1} \cdot \cdot \partial u^{\alpha_q}} \cdot H^i_{\alpha_1 \cdots \cdots \alpha_q} = 0$. The results obtained differ somewhat from those of Bortolotti and also of Kawaguchi. One fact is clear: the *natural* connection is not affine but projective. Moreover, it cannot be determined intrinsically without recourse to the second partial derivatives of the $H^i_{\alpha_1 \ldots \alpha_q}$.

R. VAIDYANATHASWAMI, University of Madras

On the space of line-elements in the calculus of variations

K. NAGABHUSHANAM, Andhra University, Waltair

On curves minimising a certain integral.

If g_{ik} be metrical ground form in Euclidean 3-space, the degenerate tensor

$$G_{ik} = x^r x^s \left(g_{rs} \, g_{ik} - g_{ri} \, g_{sk}\right)$$

is connected with the varying part of moment of inertia in the theorem of parallel axes. If we inquire into the nature of the curves minimising $\int ds$ where $ds^2 = G_{ik} \, dx^i \, dx^k$ we obtain as solution

$$\frac{x^1}{l} = \frac{x^2}{m} = \frac{x^3}{n}, \text{ (l, m, n being arbitrary constants).}$$

From physical considerations, it is shown that these straight lines through the origin which are null extremals are the only curves in the solution.

K. NAGABHUSHANAM, Andhra University, Waltair

On a class of degenerate metrics

The nature of the metrics which are necessarily degenerate, whose only extremals are null extremals forming a single family of curves is studied here.

K. NAGABHUSHANAM, Andhra University, Waltair

Variational principle in a nonkinematical case of Milne's Expanding Universe.

Milne's equations for trajectories in Expanding Universe of 4 dimensions are not derivable from a regular metric as shown by Kosambi in the Kinematical case where $G \equiv -1$. Following the same notation and methods, the tensors P^i_j, R^i_{jk} for general $G(\xi)$ are calculated and found to be as follows:

$$X^2 P^i_j = GX\,(Y\delta^i_j - \dot{x}^i \dot{x}_j) + Q_j\left(p^i\Gamma - \dot{x}^i G'\,\frac{Z}{Y}\right),$$

$$X^2 R^i_{jk} = X\,(G - G'\xi)\,(\delta^i_j \dot{x}_k - \delta^i_k \dot{x}_j) + p^i\Gamma\,(p_j\dot{x}_k - p_k\dot{x}_j) + G'Z(\delta^i_j p_k - \delta^i_k p_j),$$

where

$$Q_j = Yp_j - Z\dot{x}_j \quad ; \quad G' = \frac{dG}{d\xi}\ ; \quad G'' = \frac{d^2G}{d\xi^2}\ ;$$

$$\Gamma = -G(2+G) + G'(1+G)\,(2\xi - 1) + (G')^2\,\xi(1-\xi) - 2G''\,(1+G)\,\xi\,(1-\xi).$$

In the case when $G \equiv a$ constant $\neq -1$, it is found that if a metric exists, then

$$YA[-(1+G) + \xi\,(2+G)]$$

is also a metric, A being independent of $\dot{x}^i$. As it is not possible to find such A, it is concluded that in this case also no regular metric exists.

M. V. VAIDYANATHA SASTRI, Nizamiah Observatory

Occultations of stars observed at the Nizamiah Observatory

The occultations observed at the Nizamiah Observatory from 1927-1937 are dealt with in this paper. The method of reduction is essentially that outlined by Innes in A. J. 835 and for convenience of reference is briefly explained. The reduction of occultations yields equations of the form

$$\sigma' - \sigma = \Delta\lambda \cos\,(\rho - \chi) + \Delta\beta \sin\,(\rho - \chi)$$

where

σ' = apparent angular distance of the star from the centre of the tabular moon.

σ = apparent semi-diameter of the moon.

ρ = Direction of the moon's motion measured from the north point.

χ = Position angle of occultation measured from the north point of the moon's disc in the direction N.E.S.W.

$\Delta\lambda$ and $\Delta\beta$ are the errors in the orbital longitude and latitude of the moon.

The solution, by the method of least squares, of equations of the above type for each year gives the correction to be applied to the mean longitude and latitude of the moon.

The values for $\Delta\lambda$ as reduced from the above observations are found to be in close agreement with Brown's as reduced from a very large material.

B. R. SETH, University of Delhi

Waves in a circular Canal

No exact solution exists for the problem of the transverse oscillations of water contained in a canal of circular section. When the free surface is at the level of the axis both Lamb and Rayleigh have obtained an approximate value for the frequency of the gravest mode by using Rayleigh's method for cases in which the normal types cannot be accurately determined; but both of them have failed to notice that the constrained type assumed by them is unstable, and hence the result obtained is of not much consequence. In the present paper it is shewn that for a circular canal no exact stable type can be obtained. But, if the depth of water in the canal be small compared with its diameter, an approximate determination of the frequency can be effected. For very small depths the formula

$$\sigma = \left(\frac{g}{a}\right)^{\frac{1}{2}}\left(1+\frac{5h}{2a}\right)$$

holds good, h being the depth of water in the canal.

B. R. SETH, University of Delhi

Transverse waves in canals

In this paper it is shewn that no exact solution can be obtained for transverse oscillation in a canal whose section is a closed curve. In certain cases when the depth of water in the canal is small an approximate value of the frequency can be obtained. This value approaches $\sqrt{(g/\rho)}$ as the depth becomes very small, ρ being the radius of curvature at the lowest point.

M. ZIAUD DIN, M.A., Ph.D. (Wales), Bahawalpur

Wave mechanics of the Harmonic Oscillator and the parabolic cylinder functions

The Methods of Quantum mechanics, considering the well-known Schrodinger's Wave equation, for the Harmonic Oscillator, are used to deduce the recurrence formulae for the Parabolic Cylinder Functions.

M. RAZIUDDIN SIDDIQI, Osmania University, Hyderabad

"Matricial representation of quaternions, and its application to relativistic quantum mechanics"

In continuation of the work of Kwal and others, an attempt has been made to represent the Hamiltonian quaternions as matrices. This is done by means of three sets of four matrices each, viz. e_μ, f_μ, g_μ ($\mu = 1, 2, 3, 4$), corresponding to the three products $q\,\bar{p}$, $\bar{p}\,q$ and $q\,p$, where $\bar{p}$ is the adjoint quaternion to p. All the e, f, g, are matrices of the fourth rank, e_4 and f_4 being simply the unit matrices. A few of the relations satisfied by them are

$$e_r e_s = -e_s e_r = e_t;\ f_r f_s = -f_s f_r = f_t;\ e_r^2 = f_r^2 = -e_4 = -f_4;$$
$$g_r f_s = e_r g_s = g_t;\ g_s f_r = e_s g_r = -g_t,$$

where r, s, t denote cyclic permutations of 1, 2, 3.

$$e_\mu f_\nu = f_\nu e_\mu = g_\mu g_\nu;\ \ f_\mu g_\nu = g_\nu e_\mu;\ \ g_\mu f_\nu = e_\mu g_\nu.$$

Quaternionic functions and derivatives can easily be expressed by means of these matrices. The D'Alembertian differential operator can be linearised, and Maxwell's equations expressed in a matricial quaternionic form. This matricial representation of the group of quaternions is then seen to be identical with the group of Lorentz transformations. These transformations then lead us directly to Dirac's equation in quaternionic form.

All the variables in quantum mechanics thus become quaternionic functions of the quaternionic parameter, viz. the world point. In this relativistic quantum mechanics we get four equations of motion corresponding to each equation of Heisenberg, and four uncertainty relations corresponding to each classical relation. The generalisation is analogous to that introduced by Dirac for Schrödinger's wave-equation.

B. S. MADHAVA RAO, Engineering College, Bangalore

Biquaternions in Born's Electrodynamics.

A representation of Born's original theory by the use of biquaternions is given by Watson (*Trans. Roy. Soc. Canada, 1936*). In this paper will be found alternative representations and also generalisation to the several types of Born's Electrodynamics.

M. R. DORESAMIENGAR, University of Mysore

'A Variation of Pareto's Law'

Pareto's law of incomes has been given as $N = A/x^a$. It can also be expressed as $\frac{dN}{N} \Big/ \frac{dx}{x} = -a$. This paper points out that a has to be

necessarily greater than 2 if the statement is to be significant at all. For $a < 2$ implies frequency-distributions of incomes with infinite standard deviations. But the *simplest* of variants of Pareto's law, namely, $\frac{dN}{N} \Big/ \frac{dx}{x} = -Kx$ points to distributions with finite S.D's and invariable Lorenz Coefficients. The paper also points out the economic inferences that can be drawn from the modified Pareto's Law.

S. SUBRAMANIAN, Annamalai University

Compatibility of Fisher's tests for Index Number Formulae

Out of the seven important tests devised by Prof. Irving Fisher for Index Number Formulae, Prof. Frisch found * three incompatible with one another; but as I have pointed out elsewhere † his conclusion is rendered incorrect by his failure to take into account the limitations of the general solution he gives for his differential equations.

In the present paper the tests are examined in detail and it is shown that a formula can be found so as to satisfy them simultaneously.

* Frisch : *Jour. Amer. Statist. Assoc.* (1930) : p. 404.
† Subramanian : do. do. : p. 316.

GLEANINGS

The straight line offers a curious object for analysis. It is not for the eye a very easy form to grasp. We bend it or we leave it. Unless it passes through the centre of vision, it is obviously a tangent to the points which have analogous relations to that centre. The local signs or tensions of the points in such a tangent vary in an unseizable progression; there is violence in keeping to it, and the effect is forced. This makes the dry and stiff quality of any long straight line, which the skilful Greeks avoided by the curves of their columns and entablatures and the less economical barbarians by a profusion of interruptions and ornaments.

GEORGE SANTAYANA : *The Sense of Beauty.*

A fire mist and a planet,
  A crystal and a shell,
A jellyfish and a saurian,
  And caves where the cave-men dwell
Then a sense of law and beauty,
  And a face turned from the clod;
Some call it EVOLUTION,
  And some call it GOD.

WILLIAM HERBERT CARRUTH.

Summary of Sir Shah Sulaiman's Popular Lecture

Newton's Mechanics was designed for a Superman, who is himself at rest and can observe distant bodies instantaneously and measure absolute space and time exactly. A poor human observer is not aware of his own exact motion through space and can only employ a messenger like light who travels with a finite velocity only. Nevertheless, Newton's Law of Gravitation had signal triumphs when after explaining the motion of the moon round the earth and of the earth and the other planets round the Sun it led to the discovery of unknown planets. The position of the planet Neptune in the Heavens was calculated by two astronomers independently from its effect on Uranus, and was observed at the predicted spot. As recently as 1930, Pluto was discovered as a result of calculations made by Lowell, though it is really doubtful whether on account of its small mass it could really have been predicted on those calculations.

But in some other respects Newton's Law has undoubtedly broken down, and has been dethroned by Einstein's Theory. But Relativity cannot be regarded as the last word on the subject. Relativity in its turn has certainly failed in at least two respects. First, according to it, there is in its Universe, so to speak, some cosmic form of repulsion which increases with the distance, that is to say, bodies repel each other more if they were further apart, an unnatural result. The consequence is that everything must be scattering away and heading towards swift disintegration. The whole universe is expanding or rather exploding away at a terrific pace. It was Dr. Hubble who definitely observed this effect and laid down what is called the linear proportionality law. But Dr. Hubble himself in his Rhodes Lecture at Oxford delivered in 1936 showed that an examination of the statistical observation of the nebulae demonstrates that there is no such expansion of the Universe at all. The apparent recession of the nebulae can be explained far more satisfactorily by an inherent loss of energy as light travels through the vast space during millions of years before reaching the earth.

Secondly, Rhoyd's photographs taken on the occasion of the total solar eclipse in June 1936, show that Relativity is 100% wrong in regard to the spectral shift from the edge of the Sun.

In a few months we may know whether Relativity is at all confirmed as regards the deflection of light from stars.

Sir Sulaiman's symposium lecture was illustrated by over a dozen slides showing how his theory lay midway between Newton's law and Einstein's Relativity, and how his equation was derived without Relativity postulates.

Discussion on the Teaching of Mathematics in Schools and Colleges

Thursday, the 17th March, 1938. 10-30 A.M.

Chairman:—DR. R. P. PARANJPYE

DT. T. VIJAYARAGHAVAN (Dacca) who initiated the discussion stressed the need for continual attention to the essentials of all mathematical reasoning, namely a steady logical progress from a set of hypotheses to a conclusion.

PROF. J. A. STRANG (Lucknow) while agreeing with the previous speaker pointed out that the average student is very far from having reached any such general view-point. For many mathematics remained an affair of water-tight compartments having little in common. One of the aims of every teacher should be to break down this isolation of different branches and weld Algebra, Geometry, Calculus and the rest into a powerful weapon of thought, characterised by a single method. As regards University instruction, the historical approach was valuable as a means of arousing and maintaining the interest of the student and widening his outlook.

DR. R. SIDDIQI (Hyderabad, Deccan) suggested the appointment of a commission to enquire into the Teaching of Mathematics in Indian Schools and Colleges, and suggest improvements.

According to DR. RAM BEHARI (Delhi) the aim of teaching should be utilitarian and cultural up-to the School Leaving stage, and here, there should be no insistence on rigour. At the Degree and Post-Graduate stages, the aim should be to give a sound training in the subject maintaining a modern standard of rigour, and to give ideas about mathematical discipline. He pointed out the need for basing the study of the subject on historical background so that the students might know something of the main facts of the mathematical past, and the great "heroes" of mathematics. He also suggested that some mathematicians should use mathematical methods in the elucidation of problems in Social sciences and other fields of study, so that the subject might become more interesting and useful.

The part played by examples in the teaching of mathematics was discussed by DR. B. N. PRASAD (Allahabad) who thought that too much time was devoted to them. The primary purpose of examples was to elucidate the basic theorems and for this purpose the simplest examples were the best.

DR. A. C. BANERJI (Allahabad) considered three possible aims in teaching mathematics in the M. A. and M. Sc. classes. Preparation for competitive examinations should not be our aim. On the other hand efforts should be made to move the Public Service Commission to bring the syllabus of their examination into line with the University syllabuses. He felt that a merely cultural training could not be the *sole* aim of a mathematical education. The M. A. and M. Sc. courses should be modelled on modern and progressive lines

so as to enable the student to start research immediately after the course. As regards scholarships and fellowships, more of them should be made available and their duration should be extended beyond two years.

DR. B. R. SETH (Delhi) was of opinion that research in Mathematics demanded certain inherent qualities and talents and every one should not be encouraged to take to it. Many of those who were supposed to carry on research did so because of the stipends attached to the work.

Discussing the aims of teaching Mathematics, PROF. A. NARASINGA RAO, (Annamalainagar), said that though in the case of most of those who underwent a course in the subject the actual content of the instruction in the shape of propositions and proofs were later forgotten, there still remained an immaterial residuum in the shape of an acuter appreciation of logical relations and a maturer aesthetic sense in matters intellectual. This was indeed the most valuable result of a mathematical training and it was this which we should seek to develop.

DR. F. W. LEVI (Calcutta) was of opinion that in this country teaching was hampered by examinations. Scholarships were often given for too short a period to people who felt compelled by their scholarships to publish papers of little value. It was desirable to arrange for an interchange of scholars and lecturers of different Indian Universities.

DR. ZIAUDDIN (Bahawalpur) said that two much stress was being laid at present on pure theory. He was for greater emphasis on practical work, the use of the slide rule and facility in numerical computation including the use of small calculating machines.

PROF. D. D. KOSAMBI (Poona) read extracts from Prof. Hardy's Address to the British Association "The case against the Mathematical Tripos".

Winding up the debate, the Chairman DR. PARANJPYE said that in teaching any subject we should follow the historical method. Historically, results like Taylor's series and the Fourier's Expansion Theorem were enunciated and used before the exact conditions of their validity had been investigated. It would be going too far to say that every result should be taught along with the full minutiae of its conditions of validity and with rigorous proofs. In conclusion he remarked that the object of teaching mathematics was not to produce mathematicians but good reasoners from given data.

Names of Delegates who were present at the Conference

Abhayankar, S. A., Gwalior.
Banerji, A. C., Allahabad.
Banerji, Ekanath, Cawnpore.
Bhattacharya, D. P., Bareilly.
Dholakia, K. D., Bhavnagar.
Gupta, Hansraj, Hoshiarpur.
Kaprekar, D. R., Devlali.
Kewalramani, M. K., Karachi.
Khanna, H. L., Cawnpore.
Kosambi, D. D., Poona.
Kureishy, A. M., Aligarh.
Lall, Samuel, Lahore.
Levi, F. W., Calcutta.
Manerker, S. D.
Mehrotra, B. M., Benares.
Narasinga Rao, A., Annamalainagar.
Narlikar, V. V., Benares.
Pendse, G. G., Baroda.
Prasad, B. N, Allahabad.
Ram Behari, Delhi.
Rengaswami, K., Annamalainagar.
Sambasiva Rao, K., Waltair.
Seth, B. R., Delhi.
Seth, S. D., Cawnpore.
Shah, S. M., Aligarh.
Shaikh, A. L., Karachi.
Siddiqi, M. R., Hyderabad (Deccan).
Sivasankaranarayana Pillai, S., Annamalainagar.
Srinivasiengar, C. N., Bangalore.
Srivastava, P. L., Allahabad.
Subramaniam, S., Annamalainagar.
Sulaiman, Sir S., Delhi.
Vaidyanathaswami, R., Madras.
Varma, R. S., Cawnpore.
Venkatarayudu, T., Waltair.
Vijayaraghavan, T., Dacca.

THE INDIAN MATHEMATICAL SOCIETY

Statement of Accounts for 1936

Receipts.	Rs.	A.	P.	Expenditure.	Rs.	A.	P.
Opening Balance:							
Savings Bank	1415	10	1	*Administrative Charges:*			
Current account	195	9	6	Secretary	91	0	0
Post Office Cash Certificates ...	5600	0	0	Librarian	511	0	6
Fixed Deposit	500	0	0	Treasurer	20	0	0
Subscriptions:				Assistant Secretary	269	1	9
Life Members	350	2	0	Journal printing	1141	3	3
Ordinary Members...	554	10	0	Periodicals	406	11	0
Concession Members	735	12	0	Bank Charges	5	4	10
Subscribers	728	9	0				
Journal:				*Closing Balance:*			
Sale of journals	281	15	0	Savings Bank	2453	2	10
Sale of reprints	4	0	0	Current Account	304	13	2
Grants-in-aid:				P. O. Cash Certificates	5600	0	0
Madras University (35-36)	150	0	0	Fixed Deposit	500	0	0
Do. (36-37)	150	0	0				
Oosmania University	200	0	0				
Bombay University	200	0	0				
Annamalai University	100	0	0				
Interest on Savings Bank account ...	61	1	9				
Interest on Fixed Deposit	75	0	0				
	11,302	5	4		11,302	5	4

Checked and found correct.

28—2—1938. (Sd.) **G. A. SRINIVASAN.**

(Sd.) **L. N. SUBRAMANIAM,**

28—2—1938. *Honorary Treasurer.*

THE INDIAN MATHEMATICAL SOCIETY

Statement of Accounts for 1937

Receipts.	Rs.	A.	P.
Opening Balance:			
P. O. Cash Certificates	5600	0	0
Savings Bank	2453	2	10
Current Account	304	13	2
Fixed Deposit	500	0	0
Subscriptions:			
Life Members	250	0	0
Ordinary Members...	415	4	0
Concession Members	455	4	0
Subscribers	627	6	3
Journal:			
Sale of journals	242	8	0
Grants-in-aid:			
Bombay University	200	0	0
Oosmania University	200	0	0
Delhi University	131	2	3
Interest	61	7	1
	11,440	15	7

Expenditure.	Rs.	A.	P.
Administrative Charges:			
Secretary	49	8	3
Treasurer	10	0	0
Librarian	210	12	6
Assistant Secretary	183	15	3
Joint Editor	25	0	0
Journal printing	1109	4	0
Periodicals	536	7	0
Bank Charges	4	6	0
Closing Balance:			
Savings Bank	2352	5	9
Do.	517	2	2
Current Account	342	2	2
P. O. Cash Certificates	5600	0	0
Fixed Deposit	500	0	0
	11,440	15	7

Checked and found correct.

28—2—1938. (Sd.) **G. A. SRINIVASAN.**

(Sd.) **L. N. SUBRAMANIAM,**

28—2—1938. *Honorary Treasurer.*

INAUGURAL ADDRESS

BY

DR. R. VAIDYANATHASWAMI, *Madras University*

The Indian Mathematical Society started in 1907 for the advancement of higher mathematical study and research, by the late V. Ramaswami Aiyar, M. T. Naraniengar and a group of enthusiasts, has now had a career of some thirty years. During this period, it has performed as well as it could with the facilities at its disposal, its task of stimulating mathematical advancement, by publishing one journal which has lately become bifurcated into two, and by holding mathematical conferences once in two years or more seldom according to exigencies. In the first stages of the Society, its principal preoccupation lay in what is technically termed 'elementary geometry'; while much interesting and remarkable material can be read in the earlier numbers of the Journal, we do not find there a contact with the general spirit and the methods of modern mathematical thought. The present generation which has succeeded the generation of the founders, comprises several people with western training, and now the two journals run by the Indian Mathematical Society, as well as the journals conducted by its sister localised mathematical societies, may be said to reflect the growing assimilation of modern mathematics in this country. This is no doubt satisfactory and shews that we are not exactly stationary. But at the same time it has to be confessed that the ideal towards which we are striving is far from achievement; indeed it is not yet even in sight. This ideal is nothing less than the creative assimilation in this country of the whole of the modern scientific culture in its mathematical aspect, so that the fertile Indian mind with its untold potentialities may become seized of the new scientific situation, and resolve it in its own characteristic manner, in ways which it would be impossible to predict now. Mathematics is in a fundamental sense a master key to the scientific outlook of the present day, and the distance which separates us from our ideal may be estimated from the fact that the advanced pure and applied mathematical theories which are nowhere studied in Indian Universities well outnumber those which do receive any sort of study. At our present rate of progress, it will take a considerable period of time—certainly much more than a generation—before a

respectable level of mathematical culture can be reached in our Universities. The pace has therefore to be quickened; and the responsibility therefor devolves on the Universities, whose function on the one hand, is the active promotion of higher study and research, and on the other, the raising of the level of college instruction, so as to keep pace with the advance of research.

The explanation why Indian Universities are not as effective as they might conceivably be, in their double task of prosecution of research and its infusion into college instruction, can no doubt be sought in the troubled phase of political, social and cultural transformations through which we are passing, and the consequent stress of public activity and its diversion in various channels. But we are concerned here not with explanations, but with envisaging what has yet to be achieved, and figuring out the ways and means of doing it.

To achieve the full quickening of scientific knowledge based on the mathematical outlook, a necessary condition is the recognition of the high value of mathematical research, and of the young men in whom the research activity manifests. It is sometimes said that genius is born, not made, in such a way as to suggest that mathematical researchers are freaks of nature, endowed with an innate power of fulfilling themselves independently of social or environmental conditions. In so far as such statements may be made with the intention of disavowing responsibility for the proper utilisation and fulfilment of gifted young men, they are definitely false and misleading. Mathematics like all sciences is a social and institutional activity. Mathematical insight and sensitiveness to mathematical form, which are necessary for research, though not widely manifested, should not yet be called *rare*; we can indeed find several instances of them in every batch of students passing out from any of our Universities. The racial value of these qualities has to be appreciated and realised, and when one of these students takes up research, and proves himself capable of a new and original approach to mathematical ideas, he should be regarded as an asset to the race in the great struggle, which we vaguely indicate by the word 'civilisation'. Instead of damping his enthusiasm and putting obstacles in his path, institutional facilities should be so arranged as to draw him on from work to work, maturing and fully developing his faculties, and finally settling him in that life-work and career for which he is best fitted. That there are instances in India, where young men who have proved their worth in mathematical research have been forced to take to non-academic occupations, indicates that

a high sense of academic and human values in general, and of the value of mathematical research in particular, does not always inform the administration of the academic bodies in this country.

The Americans have developed to a high degree of perfection, their organisations of institutions for higher study, and their methods for stimulating research. In our present stage of scientific culture, we might do well in following up some of their methods. The art of administration consists in the judging of the available human material with reference to the ideal to be achieved, and the inducing it to put forth the best that it is capable of. I, for one, do not think that the best that the existing Indian mathematicians are capable of, is being got out of them. In the world of ideas, *publication* is the social symbol of final achievement; and though there are several Indian Mathematicians who have done substantial work in particular directions, we have not yet begun to write treatises or books on advanced mathematical subjects. If an academic body invites each mathematician to write a work on a prescribed subject, connected with his previous work, the carrying out of this task will not only fully develop the writer himself, but the publication of such works will be the first step in the building up of an indigenous mathematical tradition. Our own society could of course follow in the footsteps of the American Mathematical Society, and undertake this work and issue Colloquium publications, if it only possessed the necessary funds; pending the acquisition of funds, this task must be left to the Universities. The cost would not be much, and could if necessary be made up by cutting down some of the expenditure on casual lectures which leave no permanent impress behind them; on the other hand, the advantages in the direction of popularisation of higher mathematics and incitement to higher study would be considerable.

Lastly, there is a group of subjects, usually classed as applications of mathematics, in which mathematical concepts and methods are guided and controlled by the nature and requirements of the particular subject-matter concerned. Historically, Mathematics has both influenced and been influenced profoundly by such applications. Subjects of this sort—like Mathematical Economics, Statistics and the theory of Probability, and the various mathematical theories evolved with reference to situations in Modern Physics—are junctions where diverse streams of knowledge meet and mingle, and are specially sacred to scientific discipline—and for the same reason that Prayag is sacred. By immersion in such junction-subjects, the

intellect becomes purified and attains to clear vision. Universities should evince a partiality for such subjects on account of their double character, and the vivid realisations of the meaning of mathematics which they supply.

One of such junction-subjects is 'the foundations of mathematics'. The fact that it is one of the subjects which have not been taught or studied in Indian Universities, is the justification of the meagre account of it which I shall attempt here. This subject is concerned with the systematic and critical account of the ideas and processes of reasoning involved in the derivation of the concept of real number. The study of these ideas and processes of reasoning will naturally have intimate relations with Logic and Metaphysics. Our ancient Indian philosophers were generally inclined to class 'number' in the category of 'gunas' or qualities, but did not seem to have been aware of the immense possibilities which lurked in the number-concept. The different views which one can take in regard to the foundations of mathematics raise fundamental philosophic issues, and can be profitably studied not only by mathematicians, but by philosophers and students of philosophy. As a general principle, philosophy should not be taught on abstract lines from books, but in relation with the living problems of the present; and the greatest philosophical problems of the current century definitely centre round the disputes about the foundations of mathematics.

The three main directions along which the theory of the foundations of mathematics has developed are Logistic or Logicism, Formalism and Intuitionism. The first or logistic view attempts to exhibit mathematics as a branch of logic. The attitude of mind of the logistic philosopher may be described roughly thus: 'Logic is the science of reasoning i.e. it is an account of the principles involved in arriving at true conclusions, starting from true premises. Mathematics most certainly consists of true propositions about the real world, which have been arrived at by rigorous deduction from true premisses. At first sight, however, it appears to contain elements which cannot belong to logic e. g. the concept of infinity in its various forms and applications. In fact, Poincaré asserted definitely that the intuition of the unending series of natural numbers, or in other words, *The principle of mathematical induction*, is the one extralogical element in mathematics, and that any attempt at proof of this principle involves a vicious circle. But it is open to us to enlarge and revise our logic, by accepting such fundamental notions as basic principles on a par with the law of contradiction.

When we do this, mathematics could be exhibited as a branch of logic, thereby reaching the status of a science whose conclusions are true and relate to the real world.'

Thus the revision and the enlargement of the scope of traditional logic were undertaken as a preliminary to the reduction of mathematics to logic. The first step was to symbolise logic, that is, to reduce it to symbols operated by definite rules, so that all the basic principles and elements required in the process of reasoning were enumerated and taken account of. The next step was to get beyond the limitations of Aristotelian logic. The two main defects of the Aristotelian scheme are:

(1) It makes no distinction between two different kinds of relations between propositions, namely those which refer to the constituents—subject and predicate—of the proposition, and those which involve no such reference.

The theory of the latter type of relations may be described as the calculus of unanalysed propositions, or the molecular theory of propositions, and is usually treated first in books on mathematical logic under the title of Propositional or sentential calculus. The relations which are studied in this calculus are those expressed by the word *not*, and by the conjunctions *and*, *or*, *if*. If p and q are propositions, the proposition, "if p then q" is written $p \supset q$ (p implies q), and the implication $p \supset q$ is usually interpreted in the *weak* sense "not-p or q." From this weak interpretation of *if*, it follows at once that a false proposition implies every proposition and a true proposition is implied by every proposition. Symbolising *and* and *or* by $\cdot$ and $+$, the axioms of the Propositional calculus as given in *Principia Mathematica* are:

$$X + X \supset X$$
$$X \supset X + Y$$
$$X + Y \supset Y + X$$
$$(X \supset Y) \cdot \supset \cdot \{(Z + X) \supset (Z + Y)\}$$
$$\{X + (Y + Z)\} \supset \{Y + (X + Z)\}.$$

The theory of the former type of relations is met in Symbolic logic by the conceptions of *Variable* and *Propositional Function*, and the classification of propositions into *particular and universal* is achieved by introducing two quantification-symbols, namely the All-symbol and the existential symbol.

(2) The second and more serious defect of the Aristotelian logic is that it considers in effect only subject-predicate types of propositions.

It is unable to give proper account of a reasoning in which relations are involved; e.g. of the reasoning in 'If there is a son, there must be a father.' In the *Principia mathematica*, propositional functions of a single variable, answering to the scope of classical logic, are treated first, and relations, which amount in substance to propositional functions of several variables, are treated later. But in modern symbolic logic, the word predicate is often used, both in affirming an attribute of a single subject, and in asserting a relation between several subjects.

It was the first task of logistic theory to resolve the notorious contradictions in mathematical reasoning, like the paradoxes of Russell, Burali-Forti and Richard. This was done by Russell's doctrine of types, according to which the possible values of the argument of a propositional function were restricted to a certain type. But this doctrine of types does not give a good account of the situation in mathematics. In mathematics, a bounded set of real numbers has an upper bound which is also a real number; now, each real number is a class of rational numbers, and the upper bound is therefore a class of classes of rational numbers, and this though it belongs to a higher type than the real number can yet be identified with a real number. To explain this 'completeness of the real-number system', the axiom of reducibility was invented, according to which there is a predicate of the lowest grade having the same extension as a predicate of any grade. This axiom seems to run counter to the very purpose for which the doctrine of types was invented, and its logical status is dubious.

On the basis of these ideas and principles, the logistic treatise 'Principia Mathematica' claims to reduce the number and function-concepts of mathematics to pure logic. Some of the defects in the Principia scheme have been pointed out by later writers and alternatives have been proposed.

According to the formalist view, the central fact about mathematics is that mathematical symbols are capable of divergent interpretations, and must therefore be meaningless in themselves. Historically this was first realised in the case of geometry. Thus in Poincaré's picture of plane hyperbolic geometry, in which there are an infinity of non-intersecting straight lines, the word 'point' is interpreted to mean point within a circle C, and 'straight line' to mean arc of circle orthogonal to C, contained within C. With this interpretation all the axioms of the geometry are true, and naturally all the results are also true. Therefore the words 'point', 'straight line', 'inci-

dence', 'distance', 'angle' occurring in hyperbolic or any other geometry, must be regarded as *not* denoting any definite types of object in themselves. They receive of course a partial implicit specification through the relations expressed by the axioms, but otherwise the words have no content whatever. Hence also mathematical propositions have no content, and therefore, though they can be deduced from the axioms, cannot be spoken of as true in themselves.

If we develop this attitude of mind, the view at which we arrive is that mathematics is a mode of operation with symbols which have no content, i.e. which do not refer to anything in particular. Certain combinations of symbols are given beforehand called axioms; also certain rules for deriving new propositions from given propositions. All propositions which can be derived in this manner from the axioms are called *demonstrable propositions.* The whole of the formal structure may be compared to a system of signs or a language. What the significant combinations of symbols are to be, and what symbols are to represent a proposition must be decided by reference to the meaning of the symbols in a particular interpretation. These are analogous to the rules of syntax of the language. If the name 'mathematics' is restricted to the purely formal deduction of demonstrable propositions from axioms, these considerations depending on the meaning of the symbols in any interpretation must be given another name 'metamathematics' or 'Theory of Proof', or 'Theory of Deductive Systems'. In a large sense then, mathematics including both aspects may be described as the study of 'the formal structure of symbolic deductive systems'. This includes logic as well, as the propositional calculus, alone or with the calculus of functions, forms such a deductive system.

The most important structural qualities of a symbolic deductive system reposing on a set of axioms are determined by the *consistency*, *independence*, and *completeness* of the latter. A set of axioms is *consistent* if a proposition and its contradictory cannot both be deduced from them. If the set is inconsistent, then since $p. \sim p \supset q$, every proposition is demonstrable; conversely, the existence of a single refutable proposition is a proof of the consistency of the system. The set of axioms is *independent* if none of the axioms is a consequence of the remaining; lastly, the set is *complete*, if every proposition (in the language supplied by the axioms) is either demonstrable or refutable. Metamathematics is largely concerned with the investigation of these properties of axiomatic systems. The calculus of propositions is an example of an axiomatic system, and its consistency can be directly

established by the construction of refutable propositions, by a method known as 'the method of truth-values'.

More generally the Entscheidungs problem, namely the problem of deciding in a finite number of steps whether a given proposition is demonstrable or refutable or neither from a given set of axioms, is the major problem of metamathematical theory. It has been solved only for the simpler types of deductive systems; e.g. the propositional calculus, and the Aristotelian type of function-calculus i.e. calculus of propositional functions of a single variable. No general method is known by which the problem can be solved for more complicated deductive systems, in particular for Arithmetic.

The parallel postulate occupies a special position with reference to the remaining axioms of Euclidean Geometry, in that it can be neither demonstrated nor refuted from them. The remarkable result has been proved by Gödel that the Principia Mathematica deductive system, and a large class of like deductive systems are similarly *incomplete;* i.e. that there are formulae of the system which are neither demonstrable nor refutable in the system. The proof depends on the same principle as Richard's antinomy. The domain of the variables in the Principia deductive system may without loss of generality be considered to be the natural numbers. On account of the nature of the deductive process, the aggregate of demonstrable formulae must be enumerable; it will not be difficult to construct an actual scheme for enumerating them, as is done for instance in Carnap's recently published book 'The Logical Syntax of Language'. These demonstrable formulae define propositional functions of a single variable, the aggregate of which must be likewise enumerable. Let such propositional functions be enumerated and let $R(n)$ denote the nth such function. Since the domain of the variables is the totality of integers, the propositional function $R(n)$ defines a class of integers which may likewise be denoted by $R(n)$. Let K denote the class of integers n which do not belong to $R(n)$, i.e. of integers n such that $n \varepsilon R(n)$ is not demonstrable in the system. We can easily shew that the propositional function corresponding to K is constructible in the system itself; hence K is an $R(q)$. Then *the proposition* $q \varepsilon R(q)$ *is neither demonstrable nor refutable in the system.* For if it were demonstrable, it would be true and therefore q would belong to K. Therefore by definition of K, $q \varepsilon R(q)$ is not demonstrable in the system. If on the other hand it were refutable, then the negation of $q \varepsilon R(q)$ would be demonstrable; hence q does not belong to k and therefore belongs to $R(q)$. Hence both $q \varepsilon R(q)$ and its negation would be provable, which is impossible.

This theorem seems to shew that the usual formalist methods of proving consistency have their limits, and that incompleteness is the characteristic of most deductive systems. The formalist theory has done great service to mathematics through its investigation of general deductive systems and its perfecting axiomatic technique.

The differences between logistic and formalism are not serious enough to stand in the way of their being welded together; in actual practice the outlook of the mathematician partakes of the view-points of both of them. Intuitionism however stands apart from them in its insistence on certain truths of a deeper character, and its expression of them in a revolutionary and paradoxical form. To stress the dynamic and creative character of mathematics, as against the static view to which the logistic and formalistic theories point, Intuitionism asserts that the objects of mathematics do not exist in a pre-determined transcendent state of being independent of the human mind, but are actually created and shaped by the Intuition which perceives them. Hence against formalists Intuitionism would urge that mathematical science deals with concrete objects, namely the cardinal integers and the relations which arise from them, which are presented to and have been constructed by the Intuition. Against logistic which believes in the pre-eminence of logic, Intuitionism would urge that Mathematics requires no other discipline as its presupposition, not even Philosophy or Logic, but is on the other hand the presupposition for every other discipline including Philosophy and Logic; for the mathematical intuition is the primordial intuition which perceives the natural numbers, and which can hardly be distinguished from Time-consciousness itself. Intuitionism derives certain crucial doctrines from this general position, which may be described as *realistic* in contrast with the *idealistic* position of logistic and formalism. Thus it follows from the doctrine regarding the nature of mathematical objects that mathematical existence *means* or is equivalent to constructibility in intuition; it also follows that mathematical problems which have remained unsolved, resisting all attempts at solution, need not necessarily *possess* a solution. If this is accepted, we can no longer say, that every mathematical proposition must in itself be either true or false, apart from any question of its being *experienced* as true or false in our constructive intuition; in other words, we must give up the principle of the excluded middle. This result should be compared with the formalist conclusion of the existence of *incomplete* systems (Gödel's theorem); the existence of dubious propositions which is arrived at by the formalist, by the analysis of deductive systems, is

reached by the Intuitionist by a direct *tour de force*, involving the surrender of the principle of the excluded middle.

The denial of the principle of excluded middle in the case of infinite aggregates, leads to revolutionary alterations in the results of classical mathematical analysis, for example in the notions of real number, limit, and convergence of infinite sequences. Several of the fundamental theorems of classical theory of functions of a real variable have to be given up, as they cannot be proved without appealing to the principle of excluded middle. The new form of function-theory which can be arrived at is very much more complicated than the classical form.

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REVIEWS

MADHUSUDAN SARKAR and BENODE BEHARI DUTTA *Astronomy*
Revised Edition (Kamala Book Depot, Calcutta)

Price Rs. 5/-

The book under review is the outcome of an attempt by the authors to produce a suitable text-book on astronomy that will meet the requirement of college students in India preparing for University examinations. Astronomy in most Indian Universities is studied only as one of the subjects for a course in Mathematics, without much emphasis being given to the observational side of the science; and a text-book intended for such a course has necessarily to be written with this object in view. The present volume, although appearing to be only an addition to the number of text-books already available, possesses certain features which are likely to make it useful to students beginning the study of the subject. Considerable care has been exercised in the choice of material and the plan of presentation is uniformly good.

A commendable feature of the book is the inclusion of matter that usually forms part of descriptive astronomy such as the system of stars and nebulae, the spectroscope and its applications. The treatment, in this part of the book is rather too brief; and perhaps one may be permitted to remark that the descriptions are not clear in some places and the explanations have not been given with sufficient precision. A few mistakes are also met with; for example, it may be observed that in the classification of variable stars p. 265, the name β Lyrae variables seems to refer to Cepheids which form a very important class of short period variables. While explaining the phenomena of variation p. 266, no mention is made of the pulsation theory of Cepheids. The term "variable star clusters" in page 271 is apparently intended to mean the well-known "cluster variables" which are also Cepheids, a class of stars of great importance in connection with the determination of parallaxes of globular clusters and other remote objects. The word 'opticians' in the same page appears to have been used instead of 'astronomers'. There are some obvious misprints which need not be given here; it is hoped that these will be rectified in the next edition. If one may venture a suggestion, the authors may take the opportunity to revise the descriptive part of the book thoroughly and, if possible add reproductions of a few typical photographs illustrating the objects described.

The collection of questions and numerical problems given by way of exercise is sure to be helpful to the beginner in understanding the principles of the subject. The book can be recommended to college students taking a course of astronomy for degree examinations.

T. P. B.

GLEANINGS

The revolution of the corpuscle

A corpuscle once did oscillate so quickly to and fro,
He always raised disturbances wherever he did go.
He struggled hard for freedom against a powerful foe—
An atom—who would not let him go.
The aether trembled at his agitations
In a manner so familiar that I only need to say,
In accordance with Clerk Maxwell's six equations
It tickled people's optics far away.
You can feel the way it's done,
You may trace them as they run—
$d\gamma$ by dy less $d\beta$ by dz is equal $K \cdot dX/dt$
. . . .
. . . .
While the curl of (X, Y, Z) is the minus d/dt of the vector (a, b, c).

Some professional agitators only holler till they're hoarse,
But this plucky little corpuscle pursued another course,
And finally resorted to electromotive force

Resorted to electromotive force.
The medium quaked in dread anticipation,
It feared that its equations might be somewhat too abstruse,
And not admit of finite integration
In case the little corpuscle got loose,
For there was a lot of gas
Through which he had to pass,
And in case he was too rash,
There was sure to be a smash,
Resulting in a flash
Then $d\gamma$ by dy less $d\beta$ by dz would equal $K{\cdot}dX/dt$.

. . . .

. . . .

While the curl of (X, Y, Z) would be minus d/dt of the vector (a, b, c)

The corpuscle radiated until he had conceived
A plan by which his freedom might be easily achieved,
I'll not go into details, for I might not be believed
Indeed, I'm sure I should not be believed.
However, there was one decisive action.
The atom and the corpuscle each made a single charge,
But the atom could not hold him in subjection
Though something like a thousand times as large.
The corpuscle won the day
And in freedom went away
And became a cathode ray.
But his life was rather gay,
And he went at such a rate,
That he ran against a plate;
When the aether saw his fate
Its pulse did palpitate.
And $d\gamma$ by dy less $d\beta$ by dz was equal $K.dX/dt$.

. . . .

. . . .

While the curl of (X, Y, Z) was the minus d/dt of the vector (a, b, c)

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